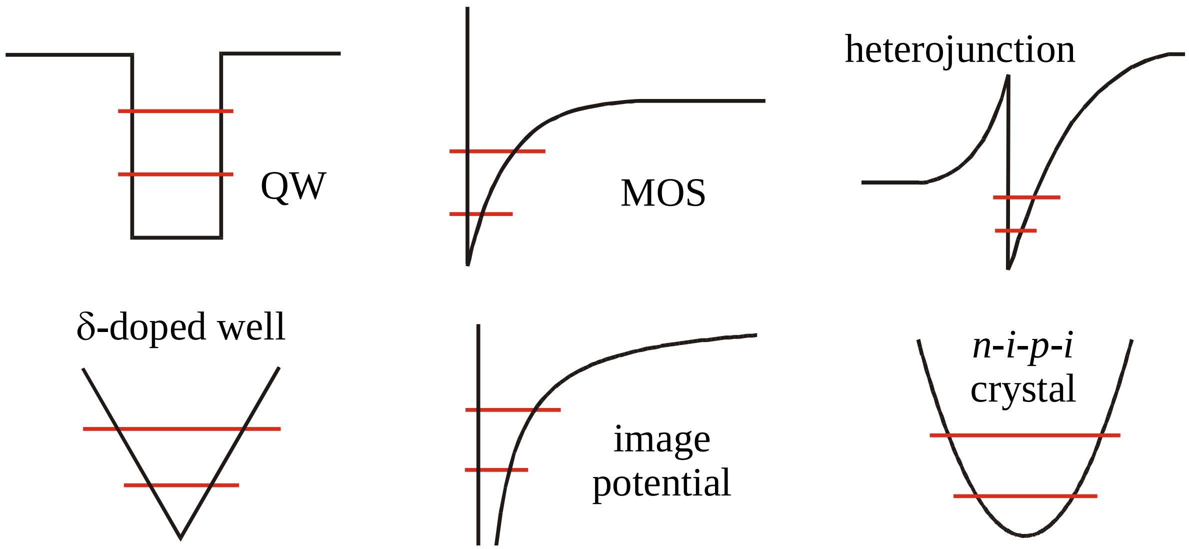
2DE: Two-dimensional electrons



$$\left[-\frac{\hbar^2}{2m}\Delta + V(z)\right]\psi(\mathbf{r}) = E\,\psi(\mathbf{r})$$

$$\psi(\mathbf{r}) = \frac{1}{L}e^{i\mathbf{k}\boldsymbol{\rho}}\varphi_n(z) \qquad E = \frac{\hbar^2 k^2}{2m} + \varepsilon_n$$

$$\mathbf{k} = (k_x, k_y, 0)$$

$$\boldsymbol{\rho} = (x, y, 0)$$

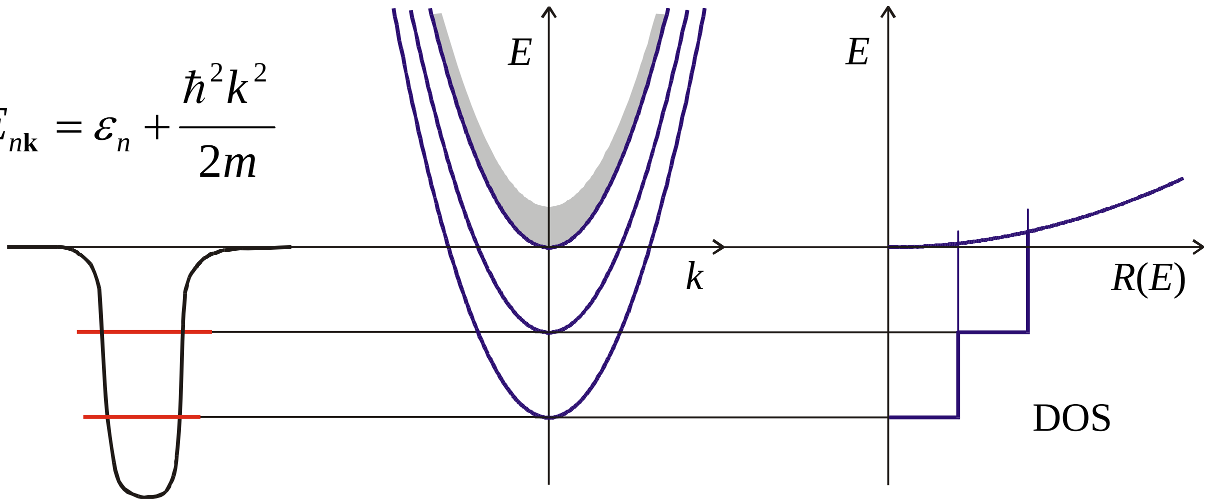
separation between levels of dimensional quantization

$$\langle \varepsilon_{\mathrm{kin}} \rangle << \Delta\varepsilon$$

kinetic energy of in-plane motion

2DE: Density of states (DOS)

$$E_{n\mathbf{k}} = \varepsilon_n + \frac{\hbar^2 k^2}{2m}$$



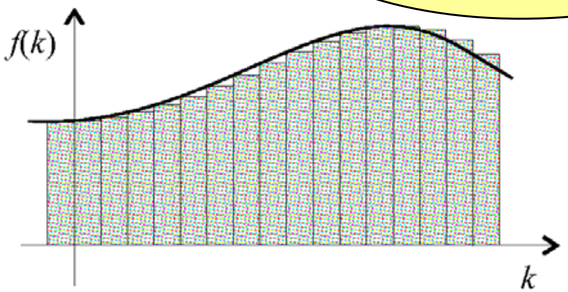
$$R(E) = \sum \delta(E - E_{n\mathbf{k}})$$

↑ DOS

$$\rho_{2\mathrm{D}} = \frac{m}{\pi \hbar^2}$$

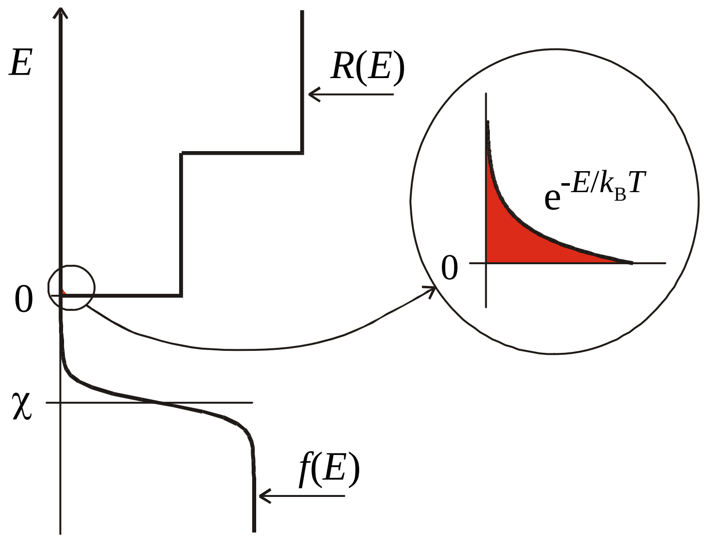
sum → integral rule

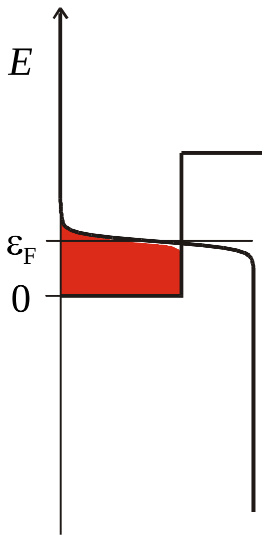
$$\sum_{\mathbf{k}} \ldots \to \left(\frac{L}{2\pi}\right)^2 \int \mathrm{d}^2 k \ldots$$



V. Karpus, "Quantum Semicondutor Structures", Lecture 4 (2010)

2DE: Bolzmann and Fermi gasses





- Boltzmann 2DEG

$$\chi < 0 , \quad |\chi| >> k_B T$$

$$n = \rho_{2D}\, k_B T\, e^{-|\chi|/k_B T}$$

$$\langle \varepsilon_{kin} \rangle = k_B T$$

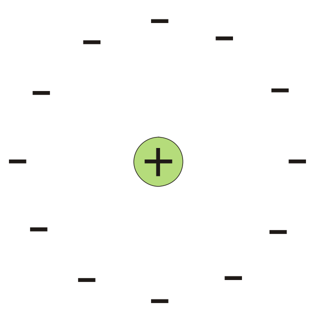
- Fermi 2DEG

$$\chi > 0 , \quad |\chi| >> k_B T$$

$$n = \rho_{2D}\, \varepsilon_F$$

$$\langle \varepsilon_{kin} \rangle = \tfrac{1}{2} \varepsilon_F$$

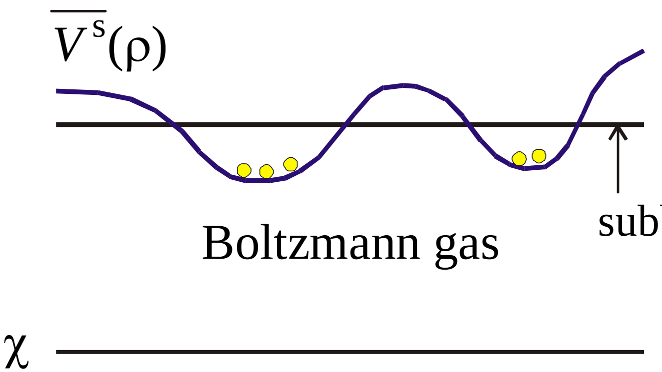
Screening

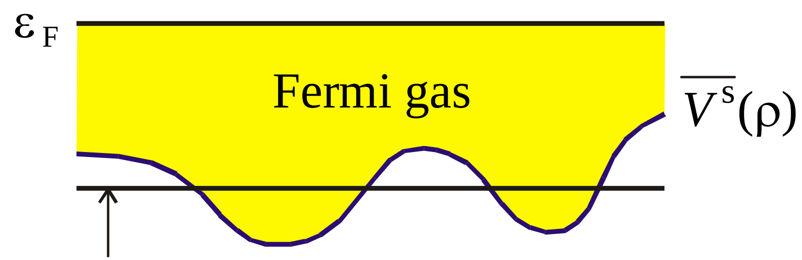


$$V^s(\mathbf{r}) = V(\mathbf{r}) - e\Phi(\mathbf{r})$$

- $V(\mathbf{r})$ — "bare" field
- $V^s(\mathbf{r})$ — screened ("dressed") field
- $e\Phi(\mathbf{r})$ — field of redistributed electrons

Thomas – Fermi scheme





$$\overline{V}^s(\boldsymbol{\rho}) = \int dz\, |\varphi_0(z)|^2 V^s(\mathbf{r})$$

$$\frac{1}{a_s} = \frac{2\pi e^2}{\kappa_0}\frac{n}{k_B T}$$

$$\frac{1}{a_s} = \frac{2}{a_B}$$

Dielectric function

Poisson equation:

$$\Delta\Phi(\mathbf{r}) = -\frac{4\pi}{\kappa_0}\rho(\mathbf{r}) \longrightarrow \frac{d^2}{dz^2}e\Phi_{q_\parallel}(z) - q_\parallel^{\,2} e\Phi_{q_\parallel}(z) = -\frac{2}{a_s}\bar{V}^{\,s}_{q_\parallel}\left|\varphi_0(z)\right|^2$$

strictly 2DE: $\varepsilon(q_\parallel) = 1 + \frac{1}{q_\parallel a_s}$

$$\bar{V}^{\,s}_{q_\parallel} = \frac{\bar{V}_{q_\parallel}}{\varepsilon(q_\parallel)} \longleftarrow \text{dielectric function}$$

quasi-2DE: $\varepsilon(q_\parallel) = 1 + \frac{H}{q_\parallel a_s}$

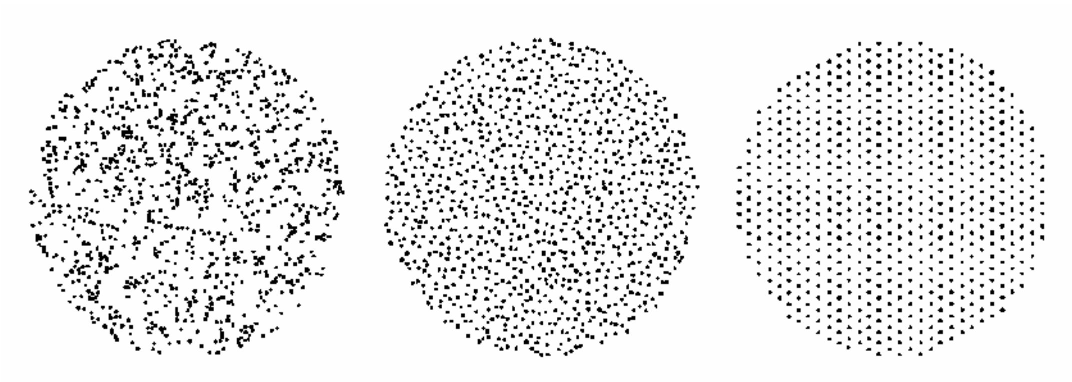
$$H = \int dz\left|\varphi_0(z)\right|^2 \int dz'\left|\varphi_0(z')\right|^2 e^{-q_\parallel |z - z'|}$$

Lindhard formula:

$$\varepsilon(q_\parallel, \omega) = 1 - \frac{4\pi e^2 H}{\kappa_0 q_\parallel}\lim_{\alpha\to 0}\int \frac{d^2 k}{(2\pi)^2}\frac{f_{\mathbf{k}} - f_{\mathbf{k}+\mathbf{q}_\parallel}}{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{q}_\parallel} + \hbar\omega + i\hbar\alpha}$$

Wigner crystal

[E. Wigner, *Phys. Rev.*, **46**, 1002 (1934)]



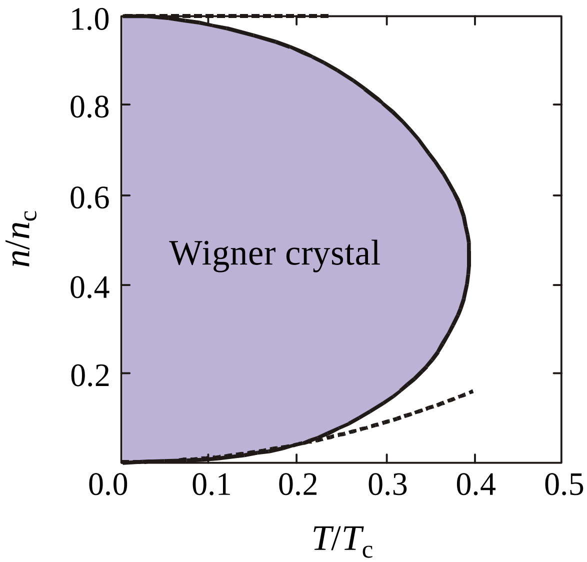
$$\Gamma = \frac{\langle V_{\mathrm{e-e}} \rangle}{\langle \varepsilon_{\mathrm{kin}} \rangle}$$

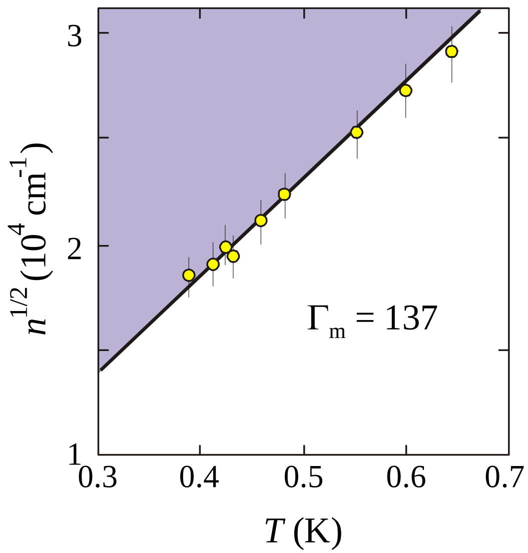
the gaseous parameter

$$\frac{n}{n_c} = \left(\frac{T}{T_c} \right)^2, \quad \text{Boltzmann 2DE}$$

$$\frac{n}{n_c} = 1, \quad \text{Fermi 2DE}$$

$$n_c = \frac{4}{\pi a_B^2 \Gamma_m^2}, \quad k_B T_c = \frac{4\varepsilon_B}{\Gamma_m^2}$$





[C.C. Grimes and G. Adams, *PRL* **42**, 795 (1979)]

Resumé

Glossary:

- DOS, density of states
- Boltzmann and Fermi 2DEG
- 2DE concentration
- screening
- Wigner crystal
- gaseous parameter

Further reading:

• (Karpus 2004, p.79-107)

Problems to solve:

•3.5.1: 0.8pt, 0.8pt, 0.5pt, 1pt

•3.5.2: 0.6pt

NB:

• sum → integral rule:

$$\sum_{\mathbf{k}} \ldots \to \left(\frac{L}{2\pi}\right)^2 \int \mathrm{d}^2 k \ldots$$

• DOS of 2DE:

$$\rho_{\mathrm{2D}} = \frac{m}{\pi \hbar^2}$$

• Fermi-Dirac function: $f = \dfrac{1}{\exp\left(\dfrac{E-\chi}{k_{\mathrm{B}}T}\right)+1}$

• math: $\int_0^\infty \mathrm{d}x\, x^{p-1} \mathrm{e}^{-x} = \Gamma(p), \quad \Gamma(p+1) = p\Gamma(p)$

$$p = m, \quad \Gamma(m+1) = m!$$

$$p = \tfrac{1}{2}, \quad \Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

• 2DE concentration:

$$n = \rho_{\mathrm{2D}}\, \varepsilon_{\mathrm{F}}$$

• screening radius:

$$a_{\mathrm{s}} = \tfrac{1}{2} a_{\mathrm{B}}$$