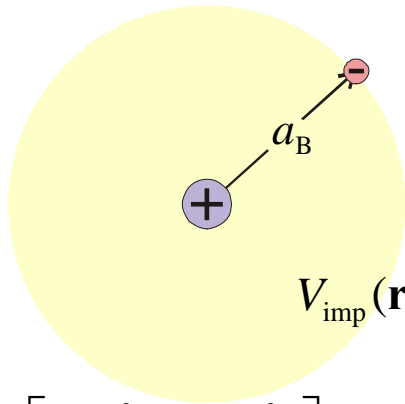


Lecture 5

Impurities and Excitons

Coulomb impurities (3D)



$$V_{\text{imp}}(\mathbf{r}) = -\frac{e^2}{\kappa_0 r}$$

$$\left[-\frac{\hbar^2}{2m} \Delta - \frac{e^2}{\kappa_0 r} \right] \psi(\mathbf{r}) = \varepsilon \psi(\mathbf{r})$$

$$\varepsilon_n = -\frac{\varepsilon_B}{n^2}, \quad n = 1, 2, \dots$$

$$\psi_{1s} = \frac{1}{\sqrt{\pi a_B^3}} \exp(-r/a_B)$$

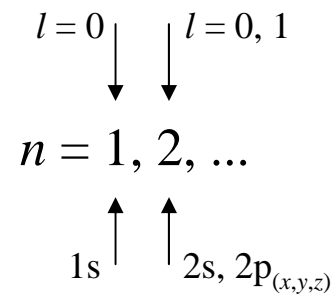
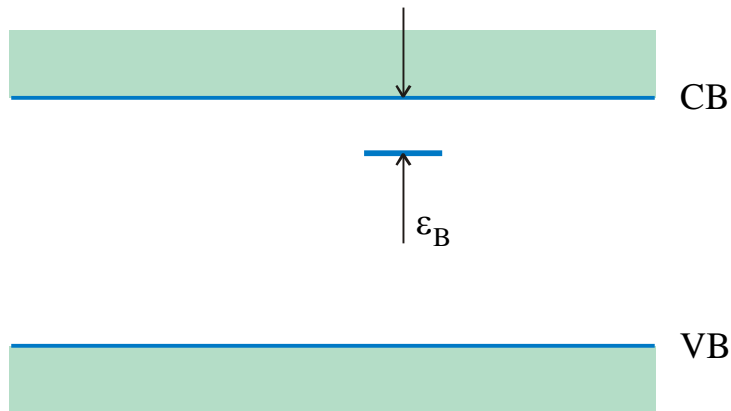
$$\varepsilon_B = \frac{me^4}{2\kappa_0^2 \hbar^2} = \text{Ry}^*$$

$$a_B = \frac{\hbar^2 \kappa_0}{me^2}$$

effective Bohr energy

effective Bohr radius

GaAs: $\kappa_0 = 12.8$, $m = 0.067 m_0 \rightarrow \varepsilon_B \approx 5.6 \text{ meV}$, $a_B \approx 100 \text{ \AA}$

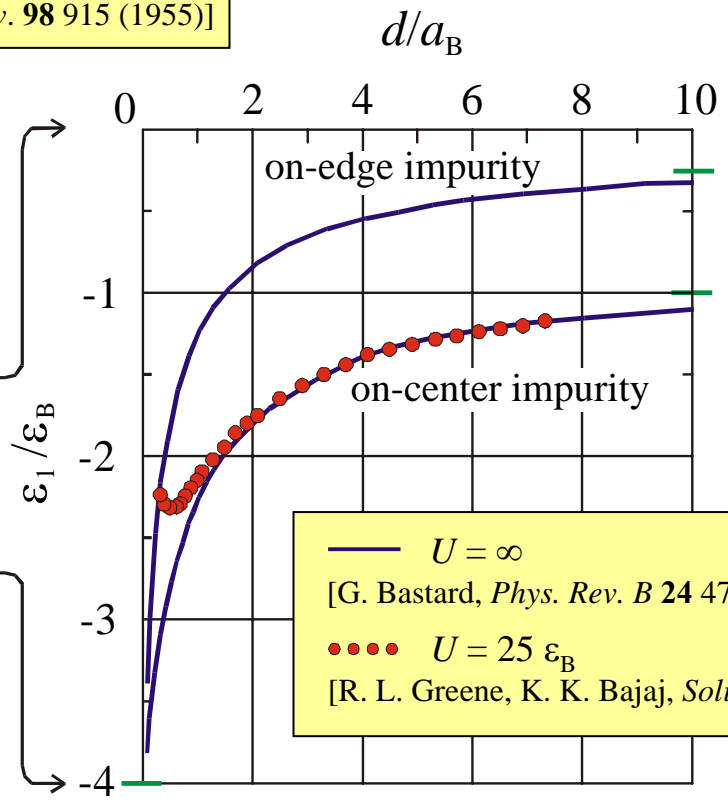
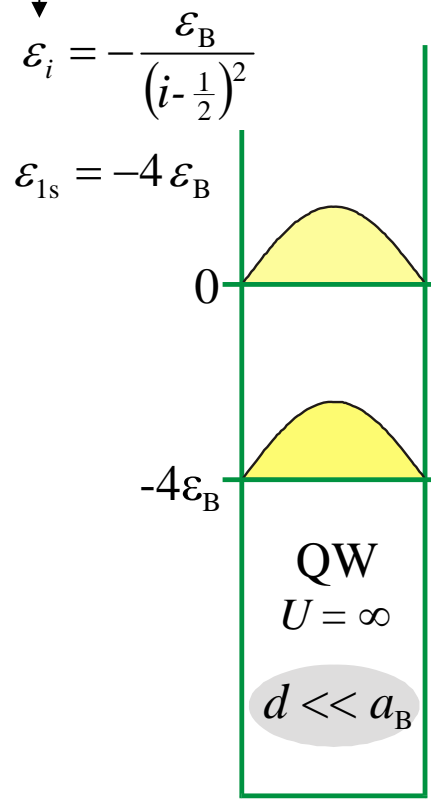


Coulomb impurities in QWs

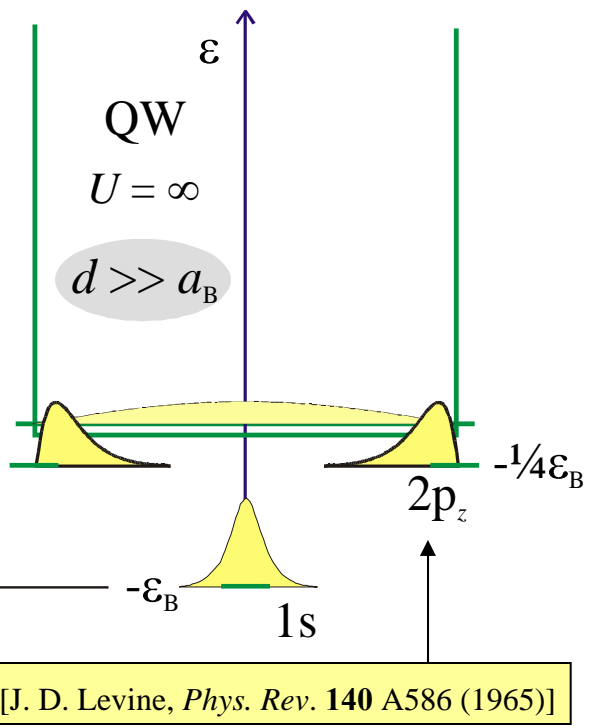
$$\left[-\frac{\hbar^2}{2m} \Delta - \frac{e^2}{\kappa_0 \sqrt{\rho^2 + (z - Z_{\text{imp}})^2}} + V(z) \right] \psi(\mathbf{r}) = \varepsilon \psi(\mathbf{r})$$

$$\varepsilon = \varepsilon(d, U; Z_{\text{imp}})$$

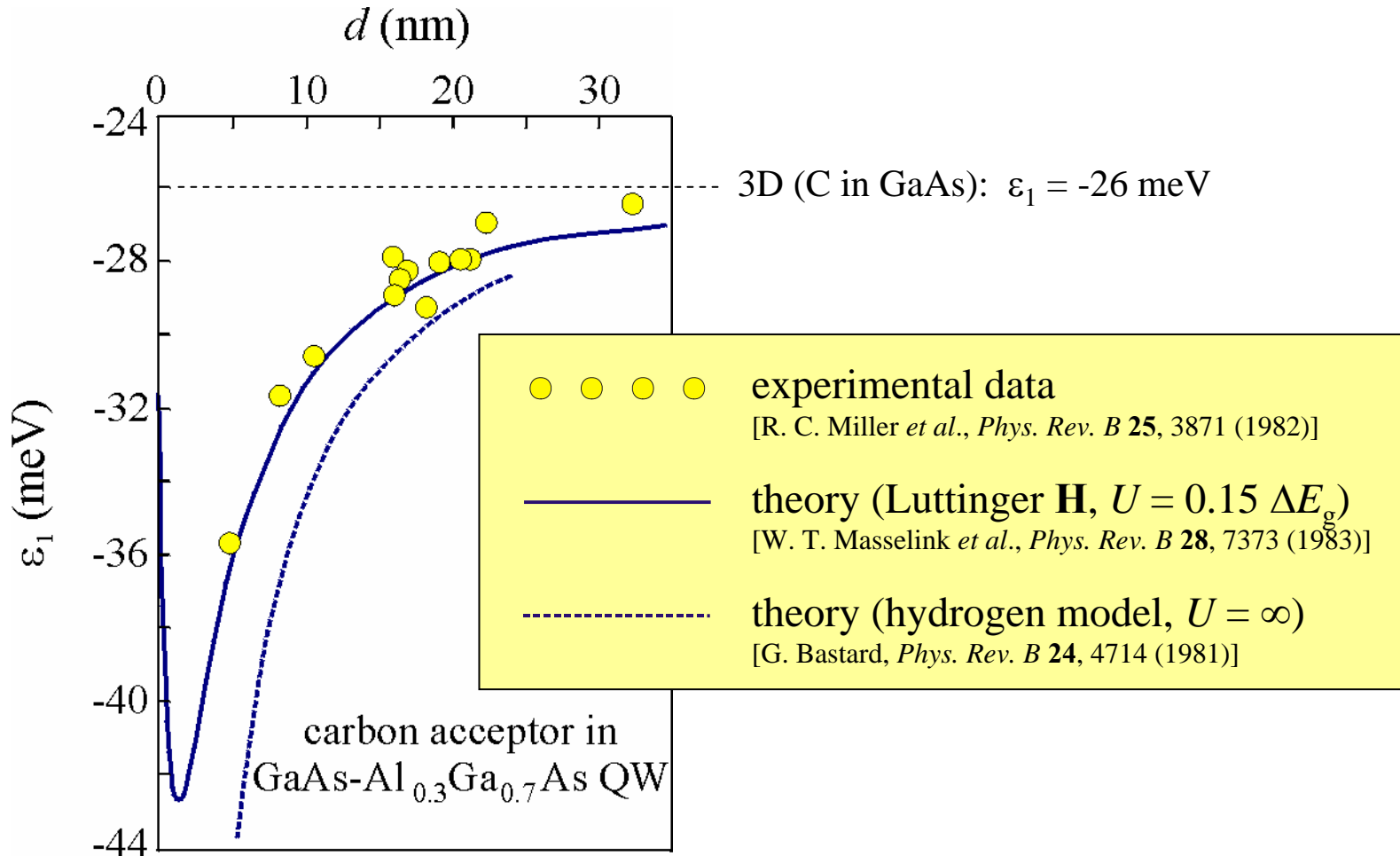
[W. Kohn, J. M. Luttinger, *Phys. Rev.* **98** 915 (1955)]



— $U = \infty$
 [G. Bastard, *Phys. Rev. B* **24** 4714 (1981)]
 ••••• $U = 25 \varepsilon_B$
 [R. L. Greene, K. K. Bajaj, *Solid State Commun.* **45** 825 (1983)]



Coulomb impurities in QWs: Experiments

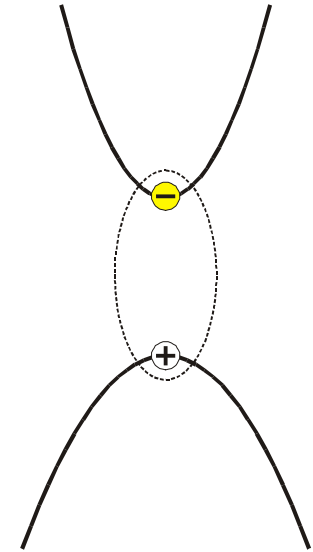
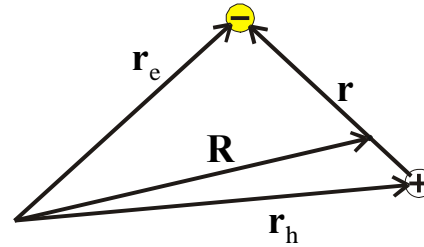


Excitons (3D)

$$\left[-\frac{\hbar^2}{2m} \Delta_e - \frac{\hbar^2}{2m_h} \Delta_h - \frac{e^2}{\kappa_0 |\mathbf{r}_e - \mathbf{r}_h|} \right] \Phi(\mathbf{r}_e, \mathbf{r}_h) = E \Phi(\mathbf{r}_e, \mathbf{r}_h)$$

$$\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$$

$$\mathbf{R} = \frac{m\mathbf{r}_e + m_h\mathbf{r}_h}{m + m_h}$$



$$-\frac{\hbar^2}{2M} \Delta_{\mathbf{R}} \psi_c = \varepsilon_c \psi_c$$

$$M = m + m_h$$

$$\left[-\frac{\hbar^2}{2\mu} \Delta_{\mathbf{r}} - \frac{e^2}{\kappa_0 r} \right] \psi = \varepsilon \psi$$

$$\frac{1}{\mu} = \frac{1}{m} + \frac{1}{m_h}$$

$$\Phi(\mathbf{r}_e, \mathbf{r}_h) = \psi_c(\mathbf{R}) \psi(\mathbf{r})$$

$$E = \varepsilon_c + \varepsilon$$

$$\frac{1}{L^{3/2}} e^{i\mathbf{K}\mathbf{R}}$$

$$\frac{\hbar^2 K^2}{2M}$$

$$\varepsilon_n = -\frac{\varepsilon_B}{n^2}$$

$$\varepsilon_B = \frac{\mu e^4}{2\kappa_0^2 \hbar^2} \quad a_B = \frac{\hbar^2 \kappa_0}{\mu e^2}$$

$$\frac{1}{m_h} = \frac{\gamma_1}{m_0} \quad [\text{A. Baldereschi, N. C. Lipari, PRB 3 439 (1971)}]$$

$$[\text{E. Kane, PRB 11 3850 (1975)}]$$

$$\text{GaAs: } \varepsilon_B \approx 4 \text{ meV}, \quad a_B \approx 140 \text{ \AA}$$

Excitons in QWs

$$\hat{H}_{\text{eh}} = -\frac{\hbar^2}{2m}\Delta_{\mathbf{r}_e} - \frac{\hbar^2}{2m_h}\Delta_{\mathbf{r}_h} - \frac{e^2}{\kappa_0|\mathbf{r}_e - \mathbf{r}_h|} + V(z_e) + V(z_h)$$

$$\begin{aligned} \mathbf{r}_e &= \boldsymbol{\rho}_e + z_e \hat{\mathbf{z}} & \mathbf{r}_h &= \boldsymbol{\rho}_h + z_h \hat{\mathbf{z}} \\ \boldsymbol{\rho} &= \boldsymbol{\rho}_e - \boldsymbol{\rho}_h & \mathbf{R}_{2D} &= \frac{m\boldsymbol{\rho}_e + m_h\boldsymbol{\rho}_h}{m + m_h} \end{aligned}$$

$$\hat{H}_{\text{eh}} = -\frac{\hbar^2}{2M}\Delta_{\mathbf{R}_{2D}} + \hat{H}$$

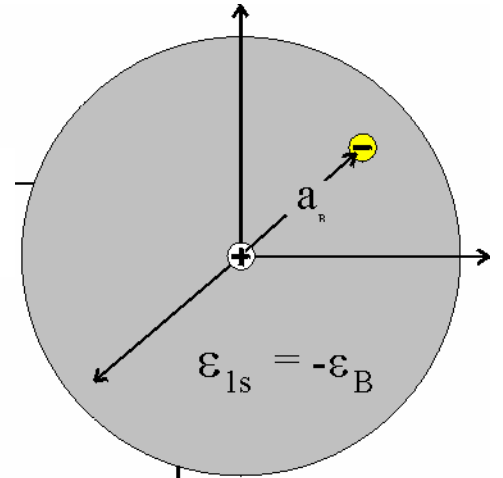
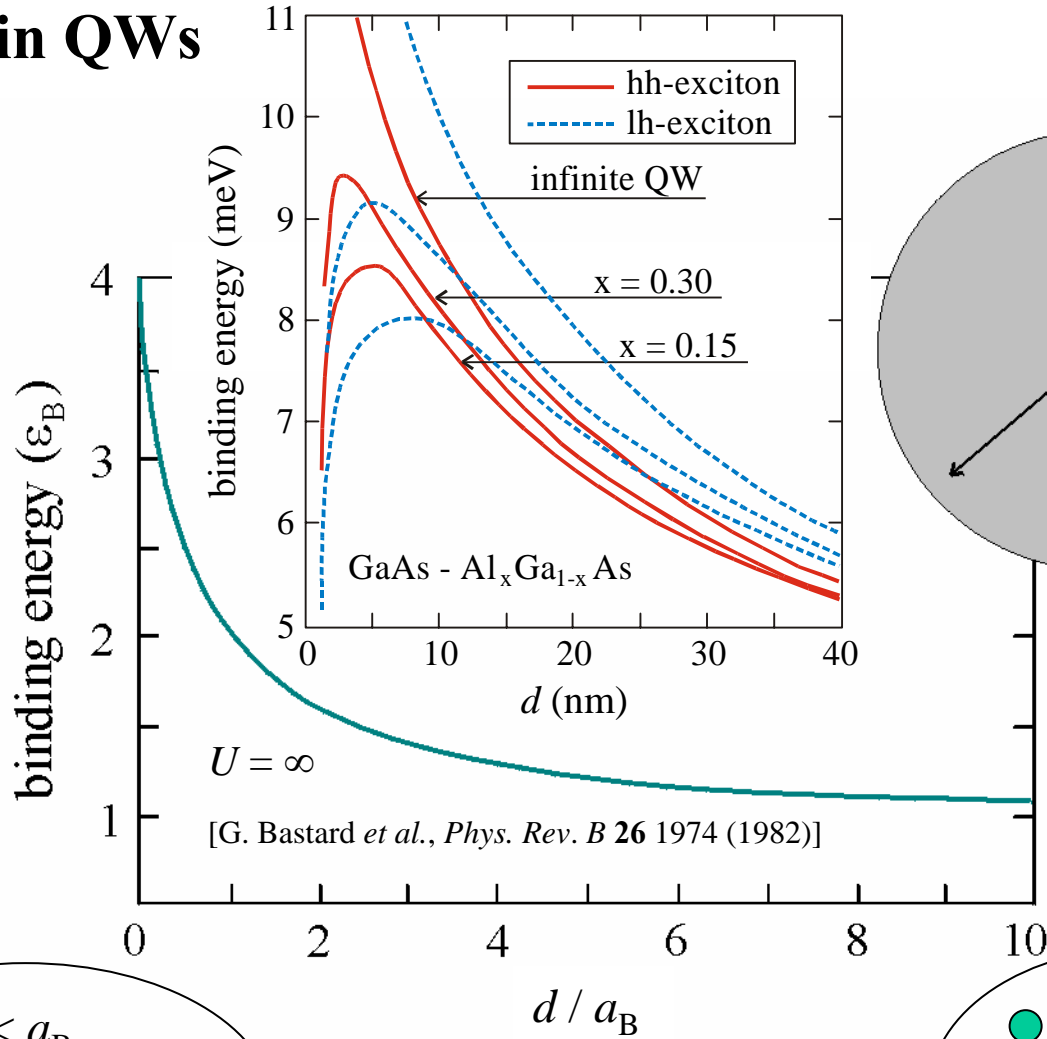
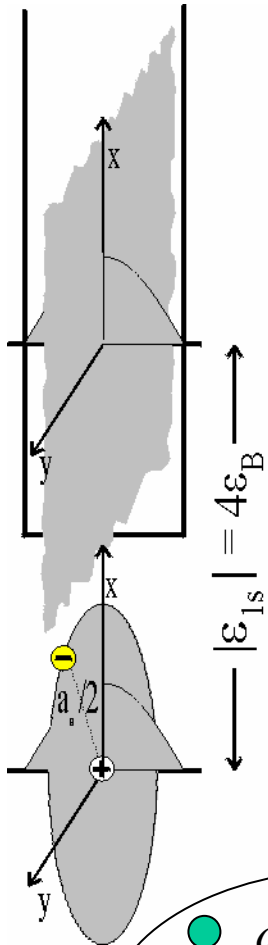
$$\hat{H} = -\frac{\hbar^2}{2\mu}\Delta_{\boldsymbol{\rho}} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z_e^2} - \frac{\hbar^2}{2m_h}\frac{\partial^2}{\partial z_h^2} - \frac{e^2}{\kappa_0\sqrt{\rho^2 + (z_e - z_h)^2}} + V(z_e) + V(z_h)$$

$$\Phi(\mathbf{r}_e, \mathbf{r}_h) = \psi_c(\mathbf{R}_{2D})\psi(\boldsymbol{\rho}, z_e, z_h) \quad E = \varepsilon_c + \varepsilon$$

$$\frac{1}{L}e^{i\mathbf{K}\mathbf{R}_{2D}} \quad \frac{\hbar^2 K^2}{2M}$$

$$\hat{H}\psi = \varepsilon\psi$$

Excitons in QWs



● $d \ll a_B$

2D exciton:

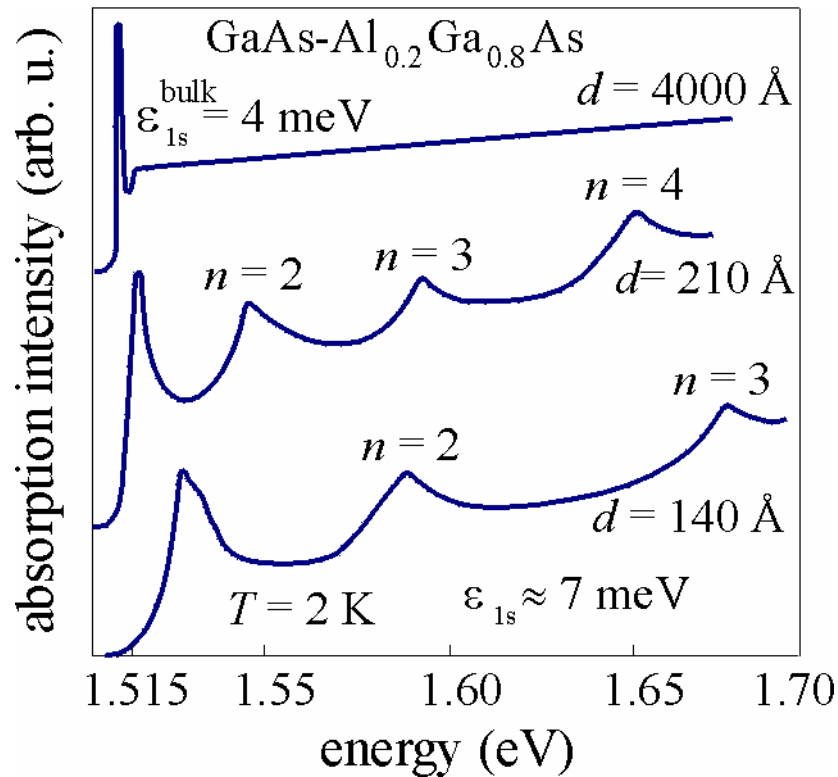
$$\epsilon_n = -\frac{\epsilon_B}{\left(n - \frac{1}{2}\right)^2}$$

● $d \gg a_B$

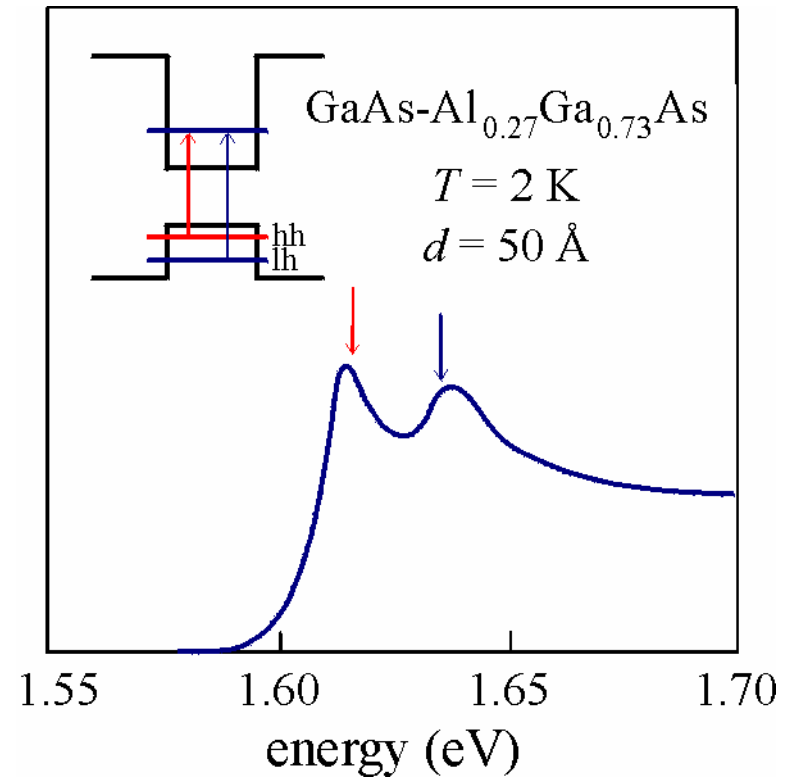
3D exciton:

$$\epsilon_n = -\frac{\epsilon_B}{n^2}$$

Excitons in QWs: Experiments

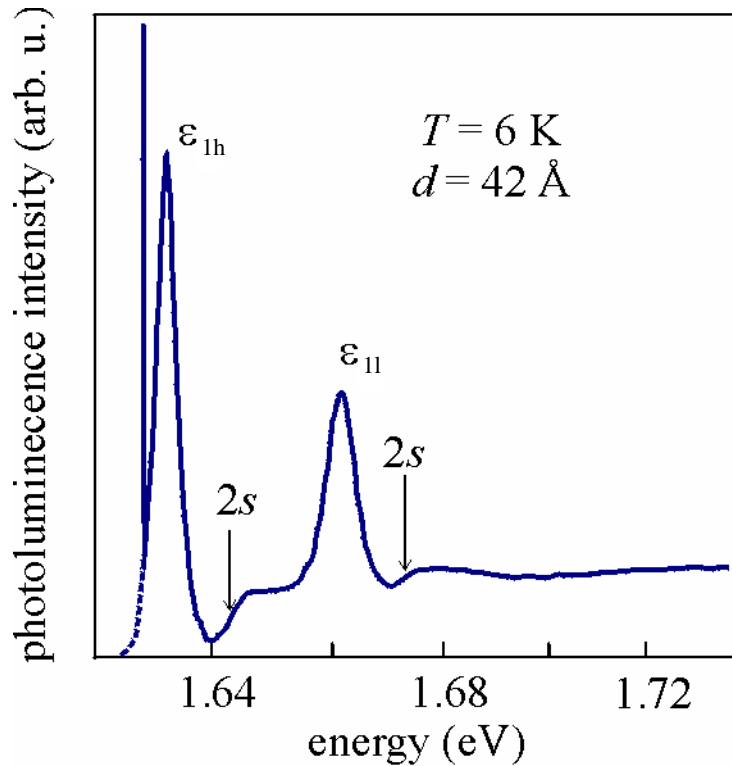


R. Dingle, W. Wiegmann, and C. H. Henry,
Phys. Rev. Lett. **33** 827 (1974)

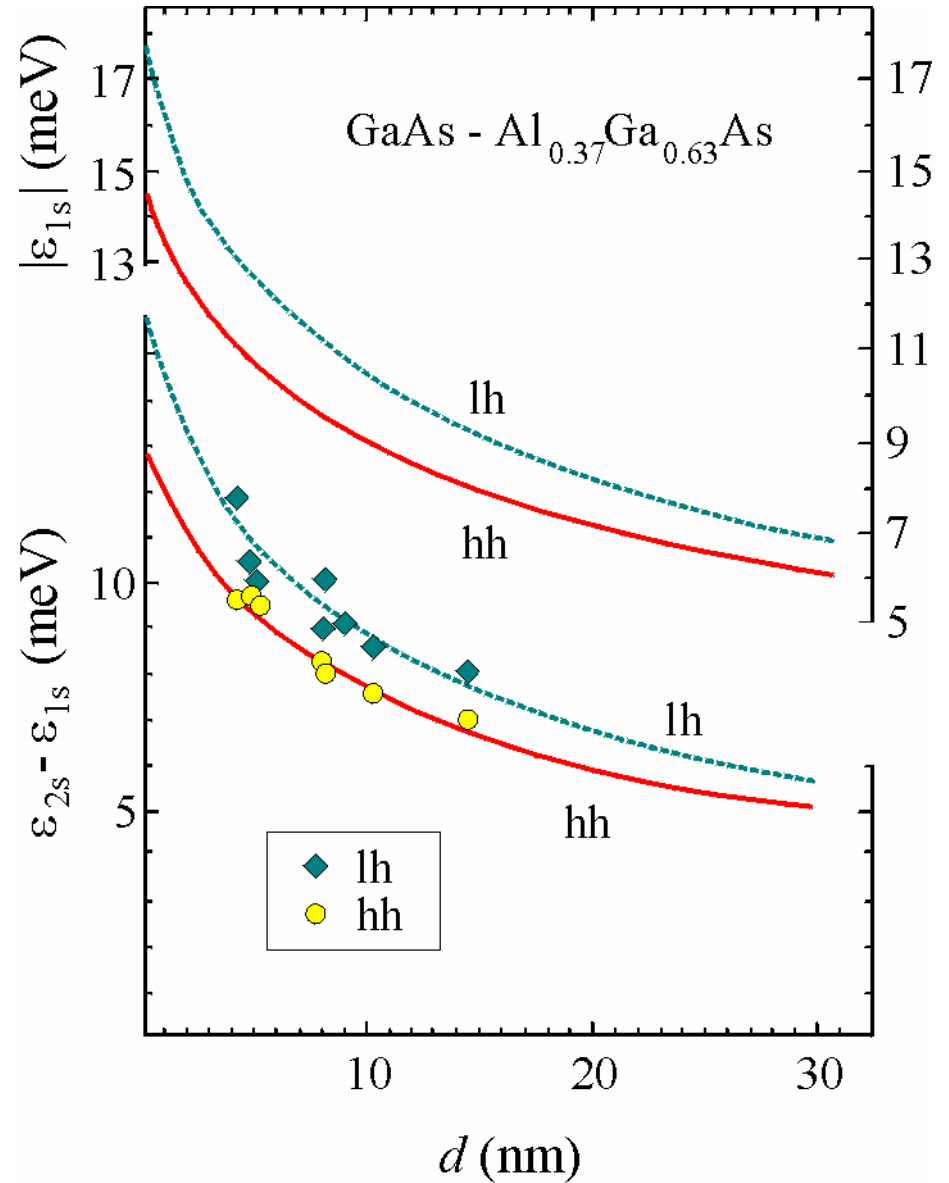


R. Dingle, A. C. Gossard, and W. Wiegmann,
Phys. Rev. Lett. **34** 1327 (1975)

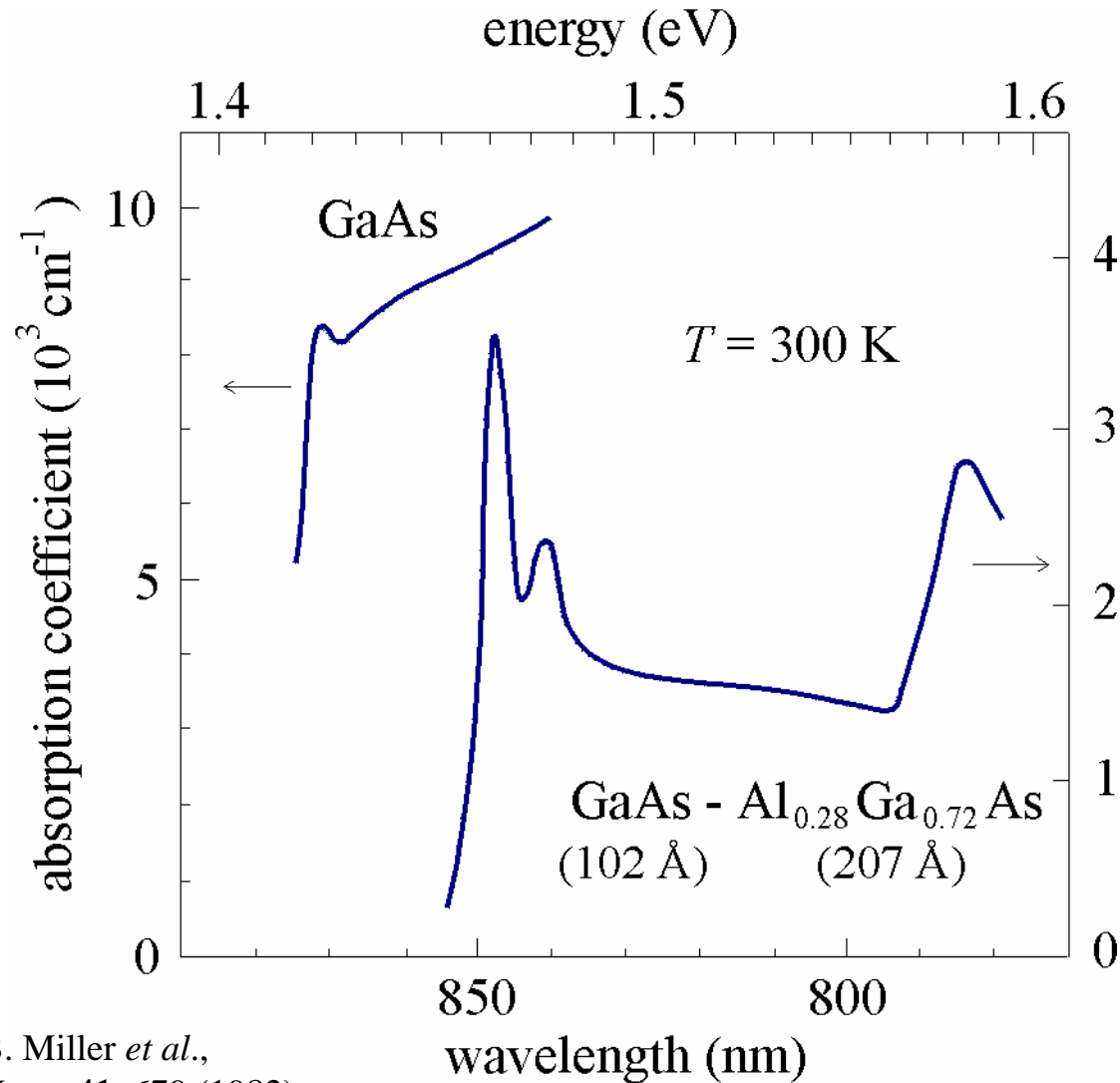
Excitons in QWs: Experiments



R. C. Miller et al., *Phys. Rev. B* **24** 1134 (1981)

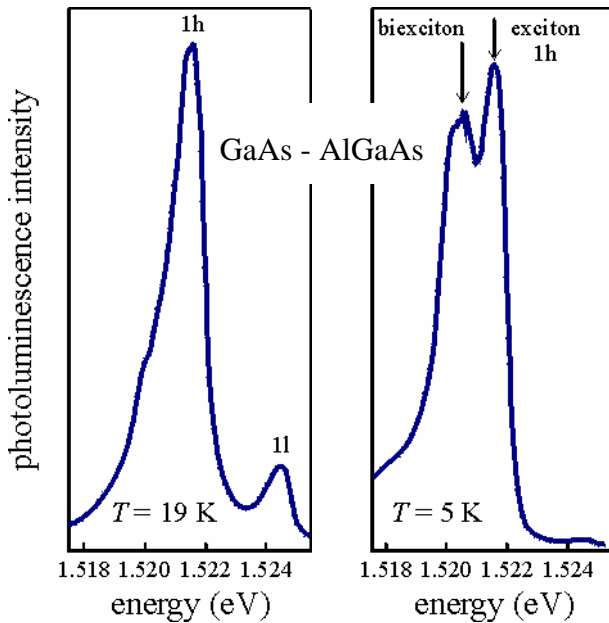
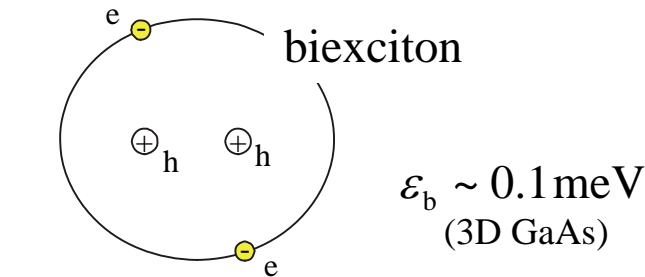


Excitons in QWs: Experiments

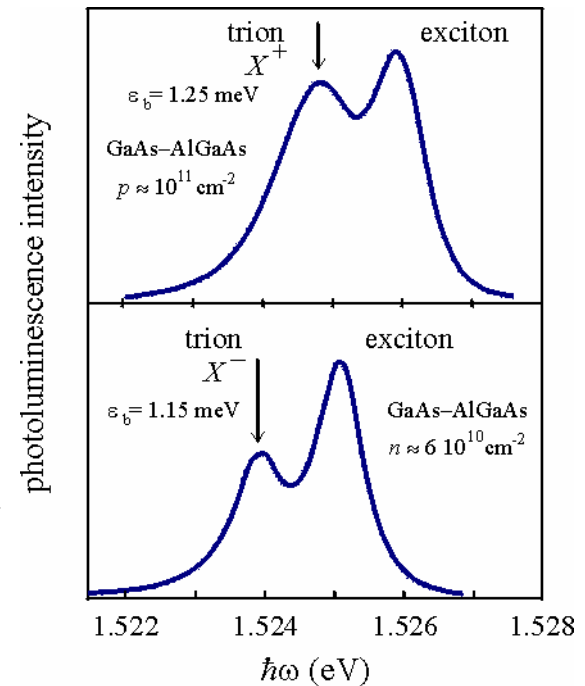
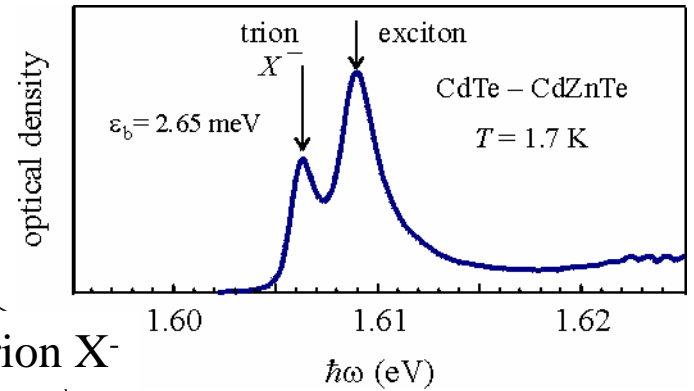
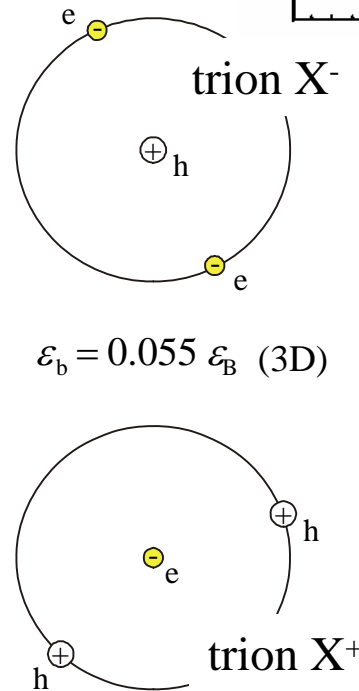


D. A. B. Miller *et al.*,
Appl. Phys. Lett., **41**, 679 (1982)

Biexcitons and trions



[R. C. Miller et al., *Phys. Rev. B* **25** 6545 (1982)]



[G. Finkelstein et al., *Phys. Rev. B* **53** R1709 (1996)]

Resumé

- Coulomb impurities
- Excitons
- Biexcitons and trions

Further reading:

- (Karpus 2004, p.109-133)

Problems to solve:

- 4.3.1: 0.6pt, 0.6pt
- 4.3.2: 1pt, 1pt

NB:

- Coulomb potential:

$$V(\mathbf{r}) = \frac{q_1 q_2}{r} \quad V(\mathbf{r}) = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

- Bohr energy and radius:

$$\epsilon_B = \frac{me^4}{2\kappa_0^2 \hbar^2} \quad a_B = \frac{\hbar^2 \kappa_0}{me^2}$$

- hydrogen spectrum:

$$\epsilon_n = -\frac{\epsilon_B}{n^2}$$

- binding energy of 2D Coulomb center:

$$\epsilon_1 = -4 \epsilon_B$$